Midterm Quiz 2016.4.27 Reference Answers

普朗克常数：$h=6.626\times10^{-34}J\cdot s$ 真空介电常数：$\varepsilon_0=8.85\times10^{-12}F/m$
基本电荷：$e=1.602\times10^{-19}C$ 玻尔磁子：$\mu_B=0.93\times10^{-23}J/T$
光速：$c=3.0\times10^{8}m/s$ 玻尔兹曼常数：$k=1.38\times10^{-23}J/K$
里德伯常数：$R_H=1.097\times10^{7}m^{-1}$ 质子质量：$m_p=1.67\times10^{-27}kg=938MeV/c^2$

一、选择题：（4 题，共 16 分，每题 4 分)

1.Using an electron with energy 12.5eV to excite the ground state of hydrogen (H) atom,the excited Hydrogen atom will transit to lower energy levels,which transitions list below can occur（ **ABD** ）

A.the 3^{rd} energy level→1^{st} level B.the 2^{nd} level→the 1^{st} level

C.the 4^{th} energy level→1^{st} level D.the 3^{rd} level→the 2^{rd} level

用能量为 12.5eV 的电子去激发处于基态的氢原子，受激发的氢原子会向低能级跃迁时发出光谱线，把氢原子由基态激发到 n=2,3,4......等能级所需要的能量为：

$$\Delta E_{1n}=E_n-E_1=-\frac{Rhc}{n^2}Z^2-\left(-\frac{Rhc}{1^2}Z^2\right)=Rhc\left(1-\frac{1}{n^2}\right)=13.6\left(1-\frac{1}{n^2}\right)eV$$

$$\Delta E_{12}=E_2-E_1=13.6\left(1-\frac{1}{2^2}\right)eV=10.2eV$$

$$\Delta E_{13}=E_3-E_1=13.6\left(1-\frac{1}{3^2}\right)eV=12.1eV$$

$$\Delta E_{14}=E_4-E_1=13.6\left(1-\frac{1}{4^2}\right)eV=12.8eV$$

可见具有 12.5eV 能量的电子不足以把基态氢原子激发到$n\geq4$的能级上去，所以只能出现$n\leq3$的能级间的跃迁。

2.Which experiments indicated the “**nuclear structure**” of atoms（ **D** ）

A.Electron double-slit experiment B.Franck-Hertz experiment

C.Stern-Gerlach experiment D.**Rutherford scattering experiment**

B.**Franck-Hertz experiment** shows when an electron collided with a mercury atom, it could lose only a specific quantity (4.9 electron volts) of its kinetic energy before flying away.It shows that there exist **stationary quantum states in atoms**.

弗兰克和赫兹在研究中发现电子与汞原子发生非弹性碰撞时能量的转移是量子化的。电子损失的能量严格地保持 4.9eV，即汞原子只接收 4.9eV 的能量。这表明原子内部结构存在稳定的量子态。

C.The **Stern-Gerlach experiment** showed that the **spatial orientation of angular momentum** of an atom in a magnetic field **is quantized**.

施特恩-盖拉赫实验表明**原子在磁场中的轨道和自旋角动量空间取向是量子化的（空间量子化）**。由电子自旋引起的磁相互作用是产生**原子精细结构**的主要因素。

D.**Rutherford scattering experiment** showed that around 1 in 8000 alpha particles were deflected by very large angles (over 90°), while the rest passed straight through with little or no deflection.Rutherford concluded that the majority of the mass was concentrated in a positively charged region (the nucleus/ central charge) surrounded by electrons.He proposed his “**nuclear model of the atom**”in 1911.

卢瑟福散射实验表明用α射线轰击金箔，绝大多数α粒子都照直穿过薄金箔，偏转很小，但大约有 1/8000 的α粒子偏转角大于 90°。与正电荷联系的质量集中在中心形成原子核，电子绕着核在核外运动。1911 年卢瑟福提出**“原子的核式结构模型”**。

3.If an atom located at $^2D_{5/2}$ state,its Lande $g-Factor$ is（ **C** ）

A. $1/6$　　B. $2/3$　　C. $6/5$　　D. $7/6$

$$^2D_{5/2} \to S=\frac{1}{2}, L=2, J=\frac{5}{2},$$

$$\boxed{g_J=\frac{3}{2}+\frac{1}{2}\left(\frac{\hat{S}^2-\hat{L}^2}{\hat{J}^2}\right)=\frac{3}{2}+\frac{1}{2}\left[\frac{S(S+1)-L(L+1)}{J(J+1)}\right]}=\frac{3}{2}+\frac{1}{2}\left[\frac{\frac{1}{2}\times\left(\frac{1}{2}+1\right)-2\times(2+1)}{\frac{5}{2}\times\left(\frac{5}{2}+1\right)}\right]=\frac{6}{5}.$$

4.The **angular momentum quantum number** of two electrons is respectively: $l_1=4, l_2=3$,thus the total orbital angular momentum quantum numbers could be（ **C** ）

A. $0,1,2,3,4$　　B. $0,1,2,3,4,5,6,7$

C. $1,2,3,4,5,6,7$　　D. $3,4,5,6,7$

假设 $\vec{L_1}, \vec{L_2}$ 分别表示是以 $l_1=4, l_2=3$ 为量子数的轨道角动量，它们的数值分别是：

$L_1=\sqrt{l_1(l_1+1)}\hbar, L_2=\sqrt{l_2(l_2+1)}\hbar$，**轨道总角动量**是这**二角动量的矢量和**，即：

$\vec{L}=\vec{L_1}+\vec{L_2}$，因为两个轨道角动量的取向是量子化的，所以合成的轨道总角动量是量子化的，它的数值为：$L=\sqrt{l(l+1)}\hbar$,并且轨道总角动量量子数 l 的取值为：

$$\boxed{l=l_1+l_2,l_1+l_2-1,\cdots,|l_1-l_2|,\because l_{\max}=l_1+l_2=7,l_{\min}=l_1-l_2=1,\therefore l=1,2,3,4,5,6,7}$$

二、填空题：（5 题，共 20 分，每题 4 分）

1.The longest wavelength photon emitted in the Balmer series(n=2) is 656.3nm .This wavelength fall in the visible spectrum Yes .

里德伯方程（氢原子光谱的经验公式）：$\tilde{\nu}=\frac{1}{\lambda}=R_H\left(\frac{1}{n^2}-\frac{1}{n'^2}\right),n=1,2,\cdots;n'>n$

巴尔末系（可见光区，眼睛可以感知的电磁波波长 $400-760nm$）：$n=2,n'=3,4,\ldots$

$$\frac{1}{\lambda}=R_H\left(\frac{1}{2^2}-\frac{1}{n'^2}\right)=R_H\cdot\frac{n'^2-2^2}{(2n')^2}\to\lambda=\frac{4n'^2}{R_H\left(n'^2-4\right)}$$

$$\to\lambda_{\max}=\frac{4\times3^2}{1.097\times10^7\times\left(3^2-4\right)}m=6.563\times10^{-7}m=656.3nm$$

巴尔末系中最著名的红色 H_α 线，$n'=3,\lambda=656.3nm$

2.The hypothesizes in Bohr’s theory are Classical Orbits with Stationary State Conditions、Frequency Condition.

玻尔的氢原子模型提出了定态条件（量子态概念）；频率条件（量子跃迁）；角动量量子化(Quantization of Angular Momentum)是依照对应原理（*correspondence principle*）的想法推出来的。

定态假设：原子系统存在一系列不连续的能量状态，处于这些状态的原子中的电子只能在一定的轨道上绕核作圆周运动，但不辐射能量，这些状态为原子系统的稳定状态，简称定态。

频率假设：当原子从能量为 E_n 的定态跃迁到另一能量为 E_k 的定态时，就要吸收或放出一个光子，频率 $\nu=\frac{|E_n-E_k|}{h}$。

轨道角动量量子化:原子中电子绕核作圆周运动的轨道角动量 $L=n\hbar,n=0,1,2\cdots$

3.An object from outer space moves past the Earth at 0.8c.You measure the length of the object as 3.3m in the Earth’s frame,its length in the object's rest frame is 5.5m .

一物体相对地球以$u=0.8c$的速度飞行，地球上的观察者测得物体为$l=3.3m$,则从物体的参考系看物体长l_0：

$$l=\frac{1}{\gamma}l_0=\sqrt{1-\frac{u^2}{c^2}}l_0\to l_0=\frac{3.3}{0.6}m=5.5m$$

4.The ground state of Al atom is $^2P_{1/2}$,its total angular momentum is $\sqrt{3}\hbar/2$,its spin magnetic moment is $-\sqrt{3}\mu_B$.

$^2P_{1/2}\to s=1/2, l=1, j=1/2$

Total angular momentum（总角动量）: $J=\sqrt{j(j+1)}\hbar=\sqrt{\frac{1}{2}\times\left(\frac{1}{2}+1\right)}\hbar=\frac{\sqrt{3}}{2}\hbar$

Spin magnetic moment（自旋磁矩）: $\mu_s=-2\sqrt{s(s+1)}\mu_B=-\sqrt{3}\mu_B$

z component of the spin magnetic moment $\mu_{s,z}$: $\mu_{s,z}=-2m_s\mu_B=\mp\mu_B$

5.The average speed of an electron in the first Bohr orbit of an atom of atomic number Z is,in units of the velocity of light αZ .

由类氢离子的电子能量公式知，在原子序数为 Z 的原子中，第一玻尔轨道上电子的平均速度（以光速 c 为单位）：

$$E_n=-\frac{1}{2}m_e(\alpha c)^2\frac{1}{n^2}Z^2\to n=1,E_1=-\frac{1}{2}m_e(\alpha cZ)^2=-\frac{1}{2}m_e v_1^2\to v_1=\alpha Z\ (c)$$

三、简答题：（4 题，共 24 分，每题 6 分）

1.What are the maximum electrons number allowed for the following quantum numbers?

（1） n,l,m_l　　（2） n,l　　（3） n

Solution:The characterization of an electron energy state in an atom needs four quantum numbers (n,l,m_l,m_s) .The **Pauli exclusion principle says:In an atom,no two electrons can have the same values for the four quantum numbers** (n,l,m_l,m_s) .**Each electron must have a unique set of these four quantum numbers.**

原子中要完全确定一个电子的能态，需要 4 个量子数:主量子数n（大体上确定原子中电子的能量）；角量子数l（确定电子的轨道角动量）；磁量子数m_l（确

定轨道角动量在外磁场方向上的分量）；自旋磁量子数m_s（确定自旋角动量在外磁场方向上的分量）。泡利不相容原理：在一个原子系统中，不可能有两个或两个以上的电子具有完全相同的四个量子数(n,l,m_l,m_s)，即，原子中的每一个状态只能容纳一个电子。

（1）n,l,m_l一定时，m_s可以有两个数值，即，当n,l,m_l一定时，原子最多可容纳 **2** 个电子；

（2）n,l一定时，m_l有$2l+1$个取值，再考虑到m_s的差别，一共有$2(2l+1)$个不同状态，即，当n,l一定时，原子最多可容纳$2(2l+1)$个电子；

（3）n一定时，l可以取$0,1,2,\cdots,n-1$，共n个不同数值，即n一定时，原子最多可容纳的电子数为：

$$N_n=\sum_{l=0}^{n-1}2(2l+1)=2+6+10+\cdots+2\left[2(n-1)+1\right]=\frac{2+(4n-2)}{2}=2n^2$$

2.Try to describe how an electron’s orbital magnetic moment interacts with an external magnetic field.

Solution:Electron’s orbital magnetic moment（电子的轨道磁矩）μ_l:

$$\vec{\mu}=-\gamma\vec{L}(1),\gamma\equiv\frac{e}{2m_e},L=\sqrt{l(l+1)}\hbar,l=0,1,2\cdots;$$

$$\rightarrow\mu_l=-\sqrt{l(l+1)}\frac{e\hbar}{2m_e}=-\sqrt{l(l+1)}\mu_B(2)$$

A magnetic moment in a homogeneous external magnetic field does not feel a force but a torque.The torque is given by（磁矩在均匀外磁场中不受力（所受合力为 0），但受到一个力矩作用，这个力矩为）：$\vec{\tau}=\vec{\mu}\times\vec{B}$（3）

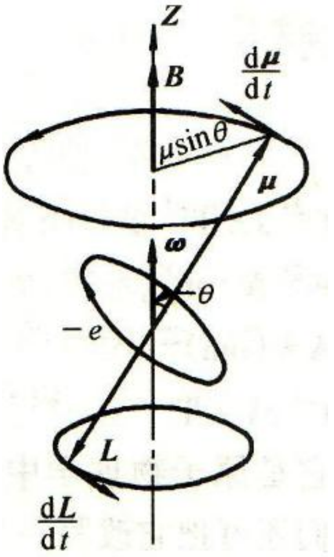
The torque will cause a change of angular momentum(*theory of angular momentum*) 由角动量定理有:$\frac{d\vec{L}}{dt}=\vec{\tau}=\vec{\mu}\times\vec{B}$（4）

From Eqs.(1)and(4),we have:$\frac{d\vec{\mu}}{dt}=-\gamma\frac{d\vec{L}}{dt}=-\gamma\vec{\mu}\times\vec{B}$（5）

The angular velocity formula for the Larmor precession（拉莫尔进动的角速度公式）：

$$\frac{d\vec{\mu}}{dt}=\vec{\omega}\times\vec{\mu},\vec{\omega}\equiv\gamma\vec{B}\text{（6）}$$

It means that in a homogeneous external magnetic field B ,a magnetic moment rotating with high speed will not line up with the field direction.Instead,it will precess about B with a certain angular velocity ω .在均匀外磁场 B 中，一个高速旋转的磁矩并不向 B 方向靠拢，而是以一定的角速度 ω 绕 B 作进动。



3.The following experiments were significant in the development of atomic physics.Choose two,in each case,briefly describe the experiment and summarize what it contributed to the development of the theory.

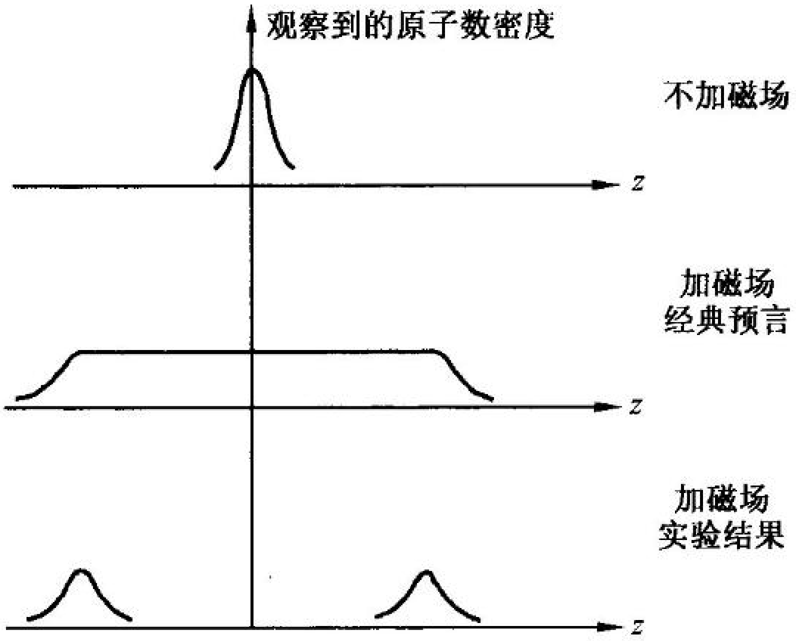
(a)Stern-Gerlach Experiment

(b)Photo-electric Effect Experiment

(c)Franck-Hertz Experiment

Solution:(a)The **Stern – Gerlach experiment** involves sending a beam of silver atoms through an **inhomogeneous magnetic field** and observing their deflection.If the atoms have a magnetic moment related to their spin angular momentum, the magnetic field gradient deflects them from a straight path. The screen reveals **discrete points of accumulation** rather than a continuous distribution, owing to the **quantum nature of spin**.The Stern-Gerlach experiment showed that the **spatial orientation of angular momentum** of an atom in a magnetic field is **quantized** and **not continuous** .

施特恩-盖拉赫实验将银原子束通过一不均匀的横向磁场并观察偏转情况。如果原子有与自旋角动量相关的磁矩（自旋磁矩），磁场梯度会使原子发生偏转，偏离原来的直线路径。由于自旋的量子性质，观察屏上看到的是离散的积累点而不是连续分布。**施特恩-盖拉赫实验表明原子在磁场中轨道和自旋角动量空间取向是量子化的（空间量子化）**。



(b)The **photo-electric effect** is the production of electrons or other free carriers when light shines upon a material. Electrons emitted in this manner can be called photo-electrons.Einstein proposed that a beam of light is not a wave propagating through space, but rather a collection of discrete wave packets (photons).He explain the photo-electric effect successfully using the light-quantum hypothesis.

在光的照射下物体发射电子或其他自由载流子的现象叫做光电效应，发射出来的电子叫做光电子。爱因斯坦假设光的能量是量子化的，光在空间的传播正像光子那样运动。他提出的光量子假说成功解释了光电效应。

(c)Franck-Hertz experiment shows when an electron collided with a mercury atom, it could lose only a specific quantity (4.9 electron volts) of its kinetic energy before flying away.It shows that there exist **stationary quantum states in atoms**.

弗兰克和赫兹在研究中发现电子与汞原子发生非弹性碰撞时能量的转移是量子化的。电子损失的能量严格地保持 4.9eV，即汞原子只接收 4.9eV 的能量。这表明原子内部结构存在稳定的量子态。

4.What are Normal and Anomalous Zeeman effects and what are their producing conditions?

Solution:**Normal Zeeman effect** is the effect that a spectral line splits into three lines with equal spacing in an external magnetic field.**Anomalous Zeeman effect** is the effect that a spectral line splits more than three lines,and the frequency intervals between components are not equal to one another.If the **net spin of the electrons is 0, or the number of electrons is even, it is called "normal Zeeman effect"**;the net spin of the electrons is not 0, It is called "Anomalous Zeeman effect".

一条谱线在外磁场作用下分裂成彼此间隔相等且间隔值为$\mu_B B$的三条谱线的现象称为正常塞曼效应；若分裂的数目不只三个且间隔也不相同则为反常塞曼效应。如果电子的总自旋$S=0$，或者说电子数目为偶数的原子才能有正常的塞曼效应，否则，总自旋不为零为反常塞曼效应。

四、计算与综合题：（共 40 分）

1.Event 1 occurs at $X_1=10m$ at $t_1=1s$. Event 2 occurs at $X_2=600000010m$ at time $t_2=2.8s$. Is there any reference frame where two events can be reversed so that event 2 occurs before event 1?Prove your answer.($12'$)

Solution:We suppose two events $(X_1,t_1),(X_2,t_2)$ occur in the reference frame K , $(X_1',t_1'),(X_2',t_2')$ occur in the reference frame K' ,the velocity of reference frame K' relative to K is u .according to the Lorentz transformation,

$$t_2'-t_1'=\frac{(t_2-t_1)-\frac{u}{c^2}(x_2-x_1)}{\sqrt{1-u^2/c^2}}$$

If event 2 occurs before event 1,that is,$t_2'-t_1'<0$

Then,we have

$$\frac{(t_2-t_1)-\frac{u}{c^2}(x_2-x_1)}{\sqrt{1-u^2/c^2}}<0$$

$$\to (t_2-t_1)-\frac{u}{c^2}(x_2-x_1)<0$$

$$\to u>\frac{(t_2-t_1)c^2}{x_2-x_1}$$

$$\to u>\frac{(2.8-1)\times\left(3.0\times10^8\right)^2}{6\times10^8}m/s=2.7\times10^8 m/s$$

That is to say,if $u > 0.9c$,event 2 can occur before event 1.

2.What is the Zeeman splitting of the D lines $3^2P_{3/2,1/2} \to 3^2S_{1/2}$ of sodium doublet at a position where the magnetic field is 2.5T?($18'$)

(1)If we look at it in the direction perpendicular to the magnetic field,how many spectra lines will be observed?

(2)If we look at it in the direction parallel to the magnetic field,how many spectra lines will be observed?What are their polarizations.

(3)Does it belong to Normal Zeeman effect?Draw a diagram for the energy levels after the splitting.

Solution:For a sodium atom in $3^2P_{3/2}$:

$$S = 1/2, L = 1, J = 3/2, m_J = \pm 3/2, \pm 1/2,$$

$$g_J = \frac{3}{2} + \frac{1}{2}\left(\frac{\hat{S}^2 - \hat{L}^2}{\hat{J}^2}\right) = \frac{3}{2} + \frac{1}{2}\left[\frac{\frac{1}{2}\times\left(\frac{1}{2}+1\right) - 1\times(1+1)}{\frac{3}{2}\times\left(\frac{3}{2}+1\right)}\right] = \frac{4}{3}$$

$$m_2 g_2 = m_J g_J = \pm 2, \pm\frac{2}{3}$$

For a sodium atom in $3^2P_{1/2}$:

$$S = 1/2, L = 1, J = 1/2, m_J = \pm 1/2,$$

$$g_J = \frac{3}{2} + \frac{1}{2}\left(\frac{\hat{S}^2 - \hat{L}^2}{\hat{J}^2}\right) = \frac{3}{2} + \frac{1}{2}\left[\frac{\frac{1}{2}\times\left(\frac{1}{2}+1\right) - 1\times(1+1)}{\frac{1}{2}\times\left(\frac{1}{2}+1\right)}\right] = \frac{2}{3}$$

$$m_1 g_1 = m_J g_J = \pm\frac{1}{3}$$

For a sodium atom in $3^2S_{1/2}$:

$$S = 1/2, L = 0, J = 1/2, m_J = \pm 1/2,$$

$$g_J = \frac{3}{2} + \frac{1}{2}\left(\frac{\hat{S}^2 - \hat{L}^2}{\hat{J}^2}\right) = \frac{3}{2} + \frac{1}{2} = 2$$

$$m_0 g_0 = m_J g_J = \pm 1$$

$$3^2P_{3/2} \to 3^2S_{1/2} : h\nu' = h\nu + \left(m_2 g_2 - m_0 g_0\right)\mu_B B = h\nu + \begin{pmatrix} \pm\frac{5}{3} \\ \pm 1 \\ \pm\frac{1}{3} \end{pmatrix} \mu_B B$$

$$\to \tilde{\nu} - \tilde{\nu}' = \frac{1}{\lambda'} - \frac{1}{\lambda} = \frac{\nu'}{c} - \frac{\nu}{c} = \frac{1}{c}\left(\nu' - \nu\right) = \begin{pmatrix} \pm\frac{5}{3} \\ \pm 1 \\ \pm\frac{1}{3} \end{pmatrix} \frac{\mu_B B}{hc} = \begin{pmatrix} \pm\frac{5}{3} \\ \pm 1 \\ \pm\frac{1}{3} \end{pmatrix} \frac{e\hbar}{2m_e} \cdot \frac{B}{2\pi\hbar c} = \begin{pmatrix} \pm\frac{5}{3} \\ \pm 1 \\ \pm\frac{1}{3} \end{pmatrix} \frac{eB}{4\pi m_e c}$$

This means that a spectral line of 5890 Å *in the sodium D lines splits into* 6 *components*

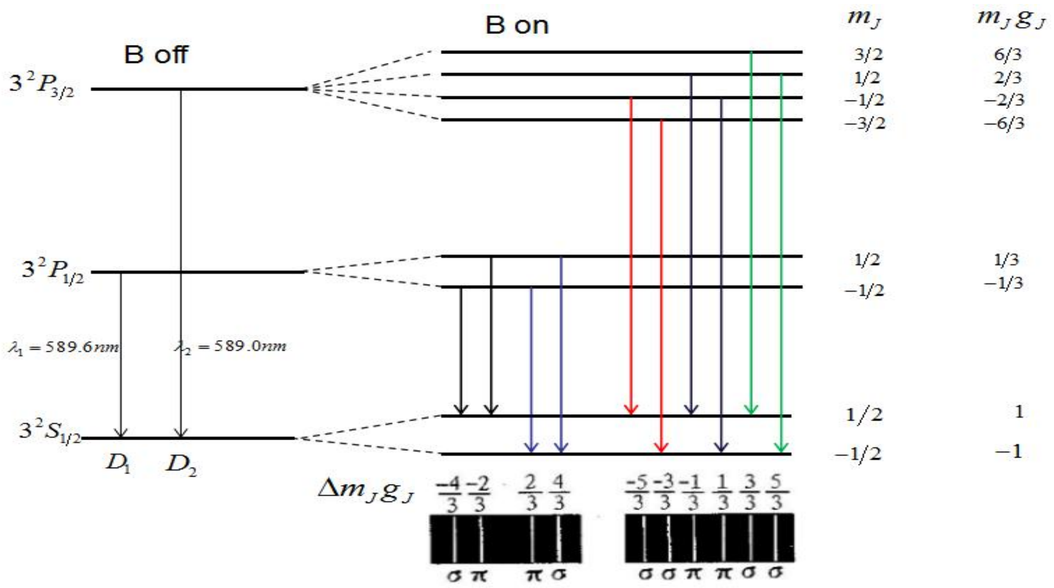
$$3^2P_{1/2} \to 3^2S_{1/2} : h\nu' = h\nu + \left(m_1 g_1 - m_0 g_0\right)\mu_B B = h\nu + \begin{pmatrix} \pm\frac{4}{3} \\ \pm\frac{2}{3} \end{pmatrix} \mu_B B$$

$$\to \tilde{\nu}' - \tilde{\nu} = \frac{1}{\lambda'} - \frac{1}{\lambda} = \frac{\nu'}{c} - \frac{\nu}{c} = \frac{1}{c}\left(\nu' - \nu\right) = \begin{pmatrix} \pm\frac{4}{3} \\ \pm\frac{2}{3} \end{pmatrix} \frac{\mu_B B}{hc} = \begin{pmatrix} \pm\frac{4}{3} \\ \pm\frac{2}{3} \end{pmatrix} \frac{e\hbar}{2m_e} \cdot \frac{B}{2\pi\hbar c} = \begin{pmatrix} \pm\frac{4}{3} \\ \pm\frac{2}{3} \end{pmatrix} \frac{eB}{4\pi m_e c}$$

This means that a spectral line of 5896 Å *in the sodium D lines splits into* 4 *components.*

$$\tilde{L} = \frac{eB}{4\pi m_e c} = \frac{1.602\times10^{-19}C \times 2.5T}{4\times3.14\times9.11\times10^{-31}kg \times 3.0\times10^{8}m/s} = 117m^{-1}$$

According to the selection rule $\Delta m = 0, \pm 1$,we draw the diagram as followed:



(a)If we look at it in the direction perpendicular to the magnetic field,**10 linearly polarized** beams are seen.**4** lines are **parallel** to B(π),the other **6** are **perpendicular** to B(σ).

(b)If we look at it in the direction parallel to the magnetic field,only 6 **circularly polarized** beams are seen,**3** are **right circularly polarization**(σ^-),the other**3** are **left circularly polarization**(σ^+)

(C)$S \neq 0$,**it doesn't belong to Normal Zeeman effect.**

3.Compute the angles between the spin angular momentum and the orbital angular momentum of the electron in $L=1$state.($10'$)

Solution:Spin angular momentum quantum number $S=1/2$,orbital angular momentum quantum number $L=1$,total angular momentum quantum number $J=L\pm S=1\pm 1/2=3/2,1/2$.

According to the Law of Cosines,the angles between the spin angular momentum and the orbital angular momentum of the electron should be:

$$\cos(S,L)=\frac{\hat{S}^2+\hat{L}^2-\hat{J}^2}{2\hat{S}\hat{L}}=\frac{S(S+1)+L(L+1)-J(J+1)}{2\sqrt{S(S+1)}\sqrt{L(L+1)}}$$

$$\cos(S,L)_1=\frac{\frac{1}{2}\times\left(\frac{1}{2}+1\right)+1\times(1+1)-\frac{3}{2}\times\left(\frac{3}{2}+1\right)}{2\times\sqrt{\frac{1}{2}\times\left(\frac{1}{2}+1\right)}\times\sqrt{1\times(1+1)}}=-\frac{\sqrt{6}}{6}$$

$$\cos(S,L)_2=\frac{\frac{1}{2}\times\left(\frac{1}{2}+1\right)+1\times(1+1)-\frac{1}{2}\times\left(\frac{1}{2}+1\right)}{2\times\sqrt{\frac{1}{2}\times\left(\frac{1}{2}+1\right)}\times\sqrt{1\times(1+1)}}=\frac{\sqrt{6}}{3}$$

$$\boxed{\rightarrow\theta(S,L)_1=\arccos\left(-\frac{\sqrt{6}}{6}\right),\theta(S,L)_2=arc\cos\left(\frac{\sqrt{6}}{3}\right)}$$